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2. (a) Use integration to find

$$\int \frac{1}{x^3} \ln x \, dx$$

(5)

(b) Hence calculate

$$\int_1^2 \frac{1}{x^3} \ln x \, dx$$

(2)



4.

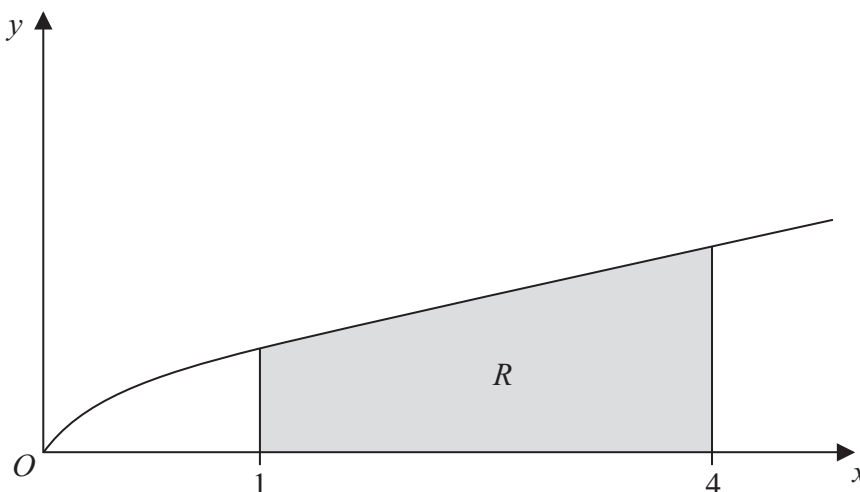


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \frac{x}{1 + \sqrt{x}}$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(a) Complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 decimal places.

(1)

x	1	2	3	4
y	0.5	0.8284		1.3333

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R , giving your answer to 3 decimal places.

(3)

(c) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of R .

(8)



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Question 4 continued

Lined area for writing the answer to Question 4.



5.

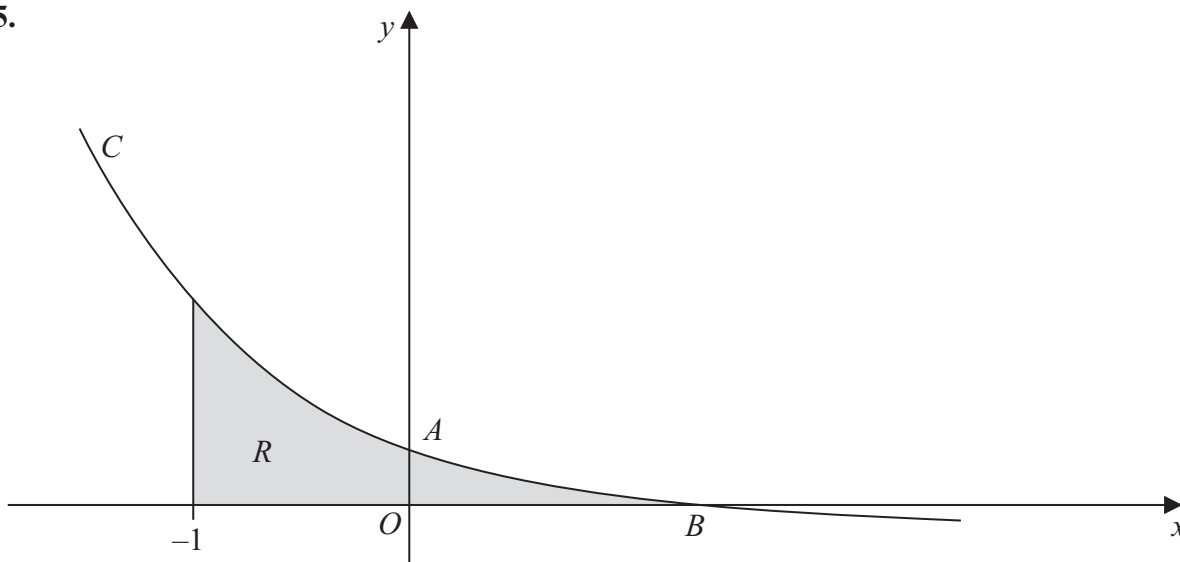


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the y -axis at the point A and crosses the x -axis at the point B .

- (a) Show that A has coordinates $(0, 3)$. (2)
- (b) Find the x coordinate of the point B . (2)
- (c) Find an equation of the normal to C at the point A . (5)

The region R , as shown shaded in Figure 2, is bounded by the curve C , the line $x = -1$ and the x -axis.

- (d) Use integration to find the exact area of R . (6)



6.

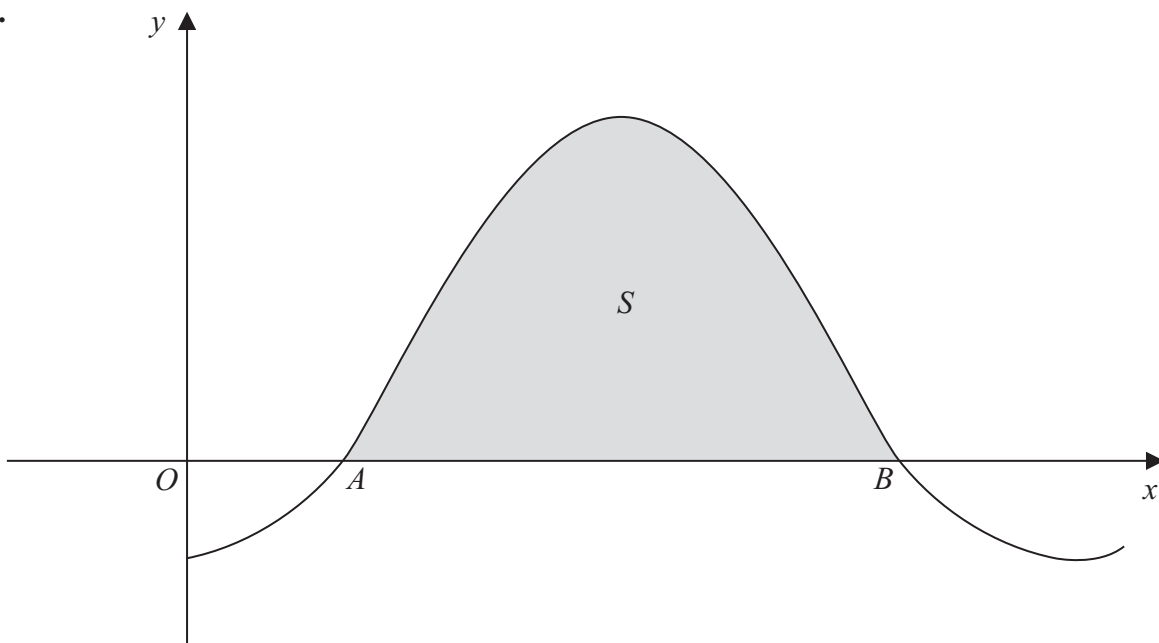


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = 1 - 2 \cos x$, where x is measured in radians. The curve crosses the x -axis at the point A and at the point B .

(a) Find, in terms of π , the x coordinate of the point A and the x coordinate of the point B . (3)

The finite region S enclosed by the curve and the x -axis is shown shaded in Figure 3. The region S is rotated through 2π radians about the x -axis.

(b) Find, by integration, the exact value of the volume of the solid generated. (6)



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7. With respect to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (9\mathbf{i} + 13\mathbf{j} - 3\mathbf{k}) + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$$

$$l_2 : \mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

where λ and μ are scalar parameters.

(a) Given that l_1 and l_2 meet, find the position vector of their point of intersection. (5)

(b) Find the acute angle between l_1 and l_2 , giving your answer in degrees to 1 decimal place. (3)

Given that the point A has position vector $4\mathbf{i} + 16\mathbf{j} - 3\mathbf{k}$ and that the point P lies on l_1 such that AP is perpendicular to l_1 ,

(c) find the exact coordinates of P . (6)



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8. A bottle of water is put into a refrigerator. The temperature inside the refrigerator remains constant at 3 °C and t minutes after the bottle is placed in the refrigerator the temperature of the water in the bottle is θ °C.

The rate of change of the temperature of the water in the bottle is modelled by the differential equation,

$$\frac{d\theta}{dt} = \frac{(3 - \theta)}{125}$$

- (a) By solving the differential equation, show that,

$$\theta = Ae^{-0.008t} + 3$$

where A is a constant.

(4)

Given that the temperature of the water in the bottle when it was put in the refrigerator was 16 °C,

- (b) find the time taken for the temperature of the water in the bottle to fall to 10 °C, giving your answer to the nearest minute.

(5)



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Question 8 continued

Lined writing area for Question 8 continued.

Q8

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

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